Modèle de Maxwell : solution exacte

Résumé :

$$f^{(0)}(\underline{V}) = \frac{n}{V_T^d} e^{-V^2/V_T^2} \frac{1}{\pi^{d/2}} \quad ; \quad \mathcal{M}(V/V_T) = \frac{1}{\pi^{d/2}} e^{-V^2/V_T^2} \frac{n}{V_T^d} \quad ; \quad V_T = \sqrt{2/\beta m} \quad : \quad f^{(0)}(\underline{V}) = \mathcal{M}(V/V_T)$$

$$(\partial_t + \underline{V}\cdot\underline{\nabla}) f(\underline{r},\underline{V};t) = p\, J_a[f,f] + (1-p)\, J_c[f,f]$$

$$J_a[f,g] = -\sigma^{d-1}\phi V_T\, g(\underline{r}, V_1;t) \int dV_2\, f(\underline{r}, V_2;t)$$

$$J_c[f,g] = \sigma^{d-1} \frac{\phi V_T}{S_d} \int dV_2 \int d\hat{\sigma}\, (b^{-1}-1)\, g(\underline{r}, V_1;t)\, f(\underline{r}, V_2;t)$$

$$\omega[f,g] = \sigma^{d-1}\phi V_T \int_{\mathbb{R}^d} dV_1\, g(\underline{r}, V_1;t) \int_{\mathbb{R}^d} dV_2\, f(\underline{r}, \underline{V}_2;t)$$

$$\zeta_n^{(0)} = n\,\sigma^{d-1}\phi V_T \quad ; \quad \zeta_n^{(0)*} = \frac{d+2}{4} \quad ; \quad \phi = \frac{4}{\sqrt{2}} \frac{\pi^{(d-1)/2}}{\Gamma(d/2)}$$

$$\zeta_T^{(0)} = \zeta_{ui}^{(0)} = 0$$

Equation de Boltzmann au premier ordre :

$$[\partial_t^{(0)} + J] f^{(1)} = -[\partial_t^{(1)} + \underline{V}\cdot\underline{\nabla}] f^{(0)}$$

$$J f^{(1)} = p\, L_a[f^{(0)}, f^{(1)}] + (1-p)\, L_c[f^{(0)}, f^{(1)}]$$

$$L_a[f^{(0)}, f^{(1)}] = -J_a[f^{(0)}, f^{(1)}] - J_a[f^{(1)}, f^{(0)}]$$

$$L_c[f^{(0)}, f^{(1)}] = -J_c[f^{(0)}, f^{(1)}] - J_c[f^{(1)}, f^{(0)}]$$

$$\Rightarrow [\partial_t^{(0)} + J] f^{(1)} = A_i \nabla_i \ln T + B_i \nabla_i \ln n + C_{ij} \nabla_i U_j + p\,\Omega f^{(1)}$$

Avec :

$$\Omega f^{(1)} = \frac{1}{n} f^{(0)} \zeta_n^{(1)} - \frac{\partial f^{(0)}}{\partial V_i} V_T \zeta_{ui}^{(1)} + \frac{\partial f^{(0)}}{\partial T} T \zeta_T^{(1)}$$

$$\zeta_n^{(1)} = \frac{2}{n}\, \omega[f^{(0)}, f^{(1)}]$$

$$= \frac{2}{n} \sigma^{d-1}\phi V_T \underbrace{\int_{\mathbb{R}^d} dV_1\, f^{(1)}(\underline{r}, \underline{V}_1;t)}_{=0} \underbrace{\int_{\mathbb{R}^d} dV_2\, f^{(0)}(\underline{r}, V_2;t)}_{=n}$$

En effet, les moments de $f^{(1)}$ sont nuls par construction même de la méthode de Chapman-Enskog : les valeurs moyennes de densité, impulsion, et énergie sont données par celles de l'état homogène $f^{(0)}$, permettant donc de définir la distribution d'équilibre local $f^{(0)}$ associée à f.

$$\zeta_T^{(1)} = -\underbrace{\zeta_n^{(1)}}_{=0} + \frac{m}{n k_B T d}\, \omega[f^{(0)}, V^2 f^{(1)}] + \frac{m}{n k_B T d}\, \omega[V^2 f^{(0)}, f^{(1)}]$$

$$= \frac{m}{n k_B T d} \sigma^{d-1}\phi V_T \left[\underbrace{\int_{\mathbb{R}^d} dV_1\, V_1^2 f^{(1)}(\underline{r}, \underline{V}_1;t)}_{=0} \underbrace{\int_{\mathbb{R}^d} dV_2\, f^{(0)}(\underline{r}, \underline{V}_2;t)}_{=n} + \underbrace{\int_{\mathbb{R}^d} dV_1\, f^{(1)}(\underline{r}, \underline{V}_1;t)}_{=0} \int_{\mathbb{R}^d} dV_2\, V_2^2 f^{(0)}(\underline{r}, \underline{V}_2;t) \right]$$

$$= 0$$

$$\zeta_{ui}^{(1)} = \frac{1}{n V_T}\, \omega[f^{(0)}, V_i f^{(1)}] + \frac{1}{n V_T}\, \omega[V_i f^{(0)}, f^{(1)}]$$

$$= \frac{1}{n V_T} \sigma^{d-1}\phi V_T \left[\underbrace{\int_{\mathbb{R}^d} dV_1\, V_{1i} f^{(1)}(\underline{r}, \underline{V}_1;t)}_{=0} \underbrace{\int_{\mathbb{R}^d} dV_2\, f^{(0)}(\underline{r}, V_2;t)}_{=n} + \underbrace{\int_{\mathbb{R}^d} dV_1\, f^{(1)}(\underline{r}, \underline{V}_1;t)}_{=0} \underbrace{\int_{\mathbb{R}^d} dV_2\, V_{2i} f^{(0)}(\underline{r}, \underline{V}_2;t)}_{=0 \text{ : symétrie } V_2 \to -V_2} \right]$$

Ainsi :

$$[\partial_t^{(0)} + J] f^{(1)} = A_i \nabla_i \ln T + B_i \nabla_i \ln n + C_{ij} \nabla_i U_j \qquad (*)$$

Coefficients de transport : méthode idem : intégration avec poids $m V_i V_j$ puis par $1/2\, m V^2$

- Intégration sur $\underline{V}$ avec poids $m V_i V_j$:

$$\underbrace{\partial_t^{(0)} \int_{\mathbb{R}^d} dV\, m V_i V_j f^{(1)}}_{= \underline{P}_{ij}^{(1)}} + \int_{\mathbb{R}^d} dV\, m V_i V_j J f^{(1)} = \underbrace{\int_{\mathbb{R}^d} dV\, m V_i V_j \underbrace{A_k(V)}_{=-A_k(-V)} \nabla_k \ln T}_{=0} + \underbrace{\int_{\mathbb{R}^d} dV\, m V_i V_j \underbrace{B_k(V)}_{=-B_k(-V)} \nabla_k \ln n}_{=0} + \int_{\mathbb{R}^d} dV\, m V_i V_j C_{k\ell}(V) \nabla_k U_\ell$$

$$\Rightarrow \boxed{\partial_t^{(0)} \underline{P}_{ij}^{(1)} + p \int_{\mathbb{R}^d} dV\, m V_i V_j L_a[f^{(0)}, f^{(1)}] + (1-p) \int_{\mathbb{R}^d} dV\, m V_i V_j L_c[f^{(0)}, f^{(1)}] = \int_{\mathbb{R}^d} dV\, m V_i V_j C_{k\ell}(V) \nabla_k U_\ell} \qquad (1)$$

Dans l'expression ci-dessus, seul le terme faisant intervenir l'opérateur d'annihilation n'a pas déjà été calculé dans la littérature. Le membre de droite se calcule avec $C_{ij}(V) = \frac{\partial}{\partial V_i}[V_j f^{(0)}] - \frac{1}{d}\frac{\partial}{\partial V_k}[V_k f^{(0)}]\,\delta_{ij}$:

$$\int_{\mathbb{R}^d} dv\, v_i v_j C_{ke}(v) = \int_{\mathbb{R}^d} dv\, v_i v_j \frac{\partial}{\partial v_k}\left[v_e f^{(0)}\right] - \frac{1}{d}\int_{\mathbb{R}^d} dv\, v_i v_j \frac{\partial}{\partial v_m}\left[v_m f^{(0)}\right]\delta_{ke}$$

$$= -\int_{\mathbb{R}^d} dv\, \underbrace{\frac{\partial}{\partial v_k}(v_i v_j)}_{= v_j \delta_{ik} + v_i \delta_{jk}} v_e f^{(0)} + \underbrace{\int_{\partial\Omega} v_i v_j v_e f^{(0)}}_{=0} + \cancel{\delta_{ke}}\frac{1}{d}\int_{\mathbb{R}^d} dv\, \underbrace{\frac{\partial}{\partial v_n}(v_i v_j)}_{= v_j \delta_{in} + v_i \delta_{jn}} v_n f^{(0)} - \frac{1}{d}\cancel{\delta_{ke}}\underbrace{\int_{\partial\Omega} v_i v_j v_n f^{(0)}}_{=0}$$

$$= -\int_{\mathbb{R}^d} dv\, v_j v_e f^{(0)}\delta_{ik} - \int_{\mathbb{R}^d} dv\, v_i v_e f^{(0)}\delta_{jk} + \frac{1}{d}\int_{\mathbb{R}^d} dv\, v_j v_n f^{(0)}\delta_{in}\delta_{ke} + \frac{1}{d}\int_{\mathbb{R}^d} dv\, v_i v_n f^{(0)}\delta_{jn}\delta_{ke}$$

$$= -\delta_{ik}\frac{n}{v_T^d}\frac{1}{\pi^{d/2}}\int_{\mathbb{R}^d} dv\, v_j v_e e^{-v^2/v_T^2} - \delta_{jk}\frac{n}{v_T^d}\frac{1}{\pi^{d/2}}\int_{\mathbb{R}^d} dv\, v_i v_e e^{-v^2/v_T^2} + \frac{1}{d}\frac{n}{v_T^d}\frac{1}{\pi^{d/2}}\delta_{in}\int_{\mathbb{R}^d} dv\, v_j v_n e^{-v^2/v_T^2}\delta_{ke}$$

$$+ \frac{1}{d}\frac{n}{v_T^d}\frac{1}{\pi^{d/2}}\delta_{jn}\int_{\mathbb{R}^d} dv\, v_i v_n e^{-v^2/v_T^2}\delta_{ke}$$

$$= -\delta_{ik}\frac{n v_T^2}{\pi^{d/2}}\int_{\mathbb{R}^d} dc\, c_j c_e e^{-c^2} - \delta_{jk}\frac{n v_T^2}{\pi^{d/2}}\int_{\mathbb{R}^d} dc\, c_i c_e e^{-c^2} + \frac{1}{d}\frac{n v_T^2}{\pi^{d/2}}\delta_{in}\int_{\mathbb{R}^d} dc\, c_j c_n e^{-c^2}\delta_{ke} + \frac{1}{d}\frac{n v_T^2}{\pi^{d/2}}\int_{\mathbb{R}^d} dc\, c_i c_n e^{-c^2}\delta_{jn}\delta_{ke}$$

$$= -\delta_{ik}\frac{n v_T^2}{\cancel{\pi^{d/2}}}\cancel{\pi^{d/2}}\frac{\cancel{d}}{2\cancel{d}}\delta_{je} - \delta_{jk}\frac{n v_T^2}{\cancel{\pi^{d/2}}}\cancel{\pi^{d/2}}\frac{\cancel{d}}{2\cancel{d}}\delta_{ie} + \frac{1}{d}\frac{n v_T^2}{\cancel{\pi^{d/2}}}\delta_{in}\cancel{\pi^{d/2}}\frac{\cancel{d}}{2\cancel{d}}\delta_{jn}\delta_{ke} + \frac{1}{d}\frac{n v_T^2}{\cancel{\pi^{d/2}}}\delta_{jn}\cancel{\pi^{d/2}}\frac{\cancel{d}}{2\cancel{d}}\delta_{in}\delta_{ke}$$

$$= n v_T^2\left[-\tfrac{1}{2}\delta_{ik}\delta_{je} - \tfrac{1}{2}\delta_{jk}\delta_{ie} + \tfrac{1}{2d}\delta_{in}\delta_{jn}\delta_{ke} + \tfrac{1}{2d}\delta_{jn}\delta_{in}\delta_{ke}\right]$$

$$= -n\frac{2}{\beta m}\left[\tfrac{1}{2}\delta_{ik}\delta_{je} + \tfrac{1}{2}\delta_{jk}\delta_{ie} - \tfrac{1}{2d}\delta_{ij}\delta_{ke} - \tfrac{1}{2d}\delta_{ij}\delta_{ke}\right]$$

$$= -n k_B T\,\frac{2}{m}\left[\tfrac{1}{2}\delta_{ik}\delta_{je} + \tfrac{1}{2}\delta_{jk}\delta_{ie} - \tfrac{1}{d}\delta_{ij}\delta_{ke}\right]$$

$$= -\underbrace{n k_B T}_{=p^{(0)}}\,\frac{1}{m}\underbrace{\left[\delta_{ik}\delta_{je} + \delta_{jk}\delta_{ie} - \tfrac{2}{d}\delta_{ij}\delta_{ke}\right]}_{:=\Delta_{ijke}}$$

$\Rightarrow$

$$\boxed{\int_{\mathbb{R}^d} dv\, m v_i v_j C_{ke}(v)\nabla_k u_e = -p^{(0)}\Delta_{ijke}\nabla_k u_e} \qquad (2)$$

Calcul de l'intégrale de collision:

$$\int_{\mathbb{R}^d} dv\, v_i v_j L_c[f^{(0)}, f^{(1)}] \overset{\text{lemme 3.4}}{=} -\sigma^{d-1}\frac{\cancel{g}v_T}{S_d}\int_{\mathbb{R}^{2d}} d\underline{v_1}d\underline{v_2}\, f^{(0)}(\underline{v_1}) f^{(1)}(\underline{v_2})\int d\hat\sigma\, \underbrace{(b-1)\left[v_{1i}v_{1j} + v_{2i}v_{2j}\right]}_{=(g\cdot\hat\sigma)\left[-g_i\sigma_j - g_j\sigma_i + 2(g\cdot\hat\sigma)\sigma_i\sigma_j\right]}$$

utilisant:

$$\int d\hat\sigma\,(\hat\sigma\cdot g)^n\sigma_i\sigma_j = \frac{\beta_n}{n+d}g^{n-2}(n g_i g_j + g^2\delta_{ij})$$

$$\int d\hat\sigma\,(\hat\sigma\cdot g)^n\sigma_i = \beta_{n+1}g^{n-1}g_i$$

$$\beta_n = \int d\hat\sigma\,(\hat\sigma\cdot g)^n = \pi^{(d-1)/2}\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n+d}{2}\right)}2$$

on trouve:

$$\int d\hat\sigma\,(g\cdot\hat\sigma)\left[-g_i\sigma_j - g_j\sigma_i + 2(g\cdot\hat\sigma)\sigma_i\sigma_j\right] = -\int d\hat\sigma\,(g\cdot\hat\sigma)g_i\sigma_j - \int d\hat\sigma\,(g\cdot\hat\sigma)g_j\sigma_i + 2\int d\hat\sigma\,(g\cdot\hat\sigma)^2\sigma_i\sigma_j$$

$$= -g_i\beta_2 g^0 g_j - g_j\beta_2 g^0 g_i + 2\frac{\beta_2}{d+2}g^0(2 g_i g_j + g^2\delta_{ij})$$

$$= -2\beta_2 g_i g_j + 2\beta_2\frac{1}{d+2}(2 g_i g_j + g^2\delta_{ij})$$

$$= -\frac{2\beta_2}{d+2}\left[(d+2)g_i g_j - 2 g_i g_j - g^2\delta_{ij}\right]$$

$$= -\frac{2\beta_2}{d+2}d\left(g_i g_j - \frac{1}{d}g^2\delta_{ij}\right) \quad ; \beta_2 = 2\pi^{(d-1)/2}\frac{\Gamma(3/2)}{\Gamma\left(\frac{d+2}{2}\right)} = 2\pi^{(d-1)/2}\frac{\frac{1}{2}\Gamma(1/2)}{\frac{d}{2}\Gamma(d/2)}$$

Ainsi:

$$\int_{\mathbb{R}^d} dv\, v_i v_j L_c[f^{(0)}, f^{(1)}] = -\sigma^{d-1}\frac{\phi v_T}{S_d}\left(-\frac{2\beta_2}{d+2}d\right)\int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\, f^{(0)}(v_1) f^{(1)}(v_2)\left(g_i g_j - \frac{1}{d} g^2 \delta_{ij}\right)$$

$$= \sigma^{d-1} 2\frac{\phi v_T}{\cancel{S_d}}\frac{\cancel{d}}{d+2}\frac{2\pi^{d/2}}{\Gamma(d/2)}\Gamma(1/2)\cancel{\pi^{-1/2}}\cancel{\frac{1}{d}}\int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\, f^{(0)}(v_1) f^{(1)}(v_2)\left(g_i g_j - \frac{1}{d} g^2 \delta_{ij}\right)$$

$$= \sigma^{d-1} 2\frac{\phi v_T}{d+2}\int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\, f^{(0)}(v_1) f^{(1)}(v_2)\left[v_{1i}v_{1j} - \overset{0}{\cancel{v_{1i}}v_{2j}} - \overset{0}{v_{2i}\cancel{v_{1j}}} + v_{2i}v_{2j} - \frac{1}{d}\delta_{ij}\left(v_1^2 + v_2^2 - 2\overset{0}{\cancel{v_{1k}}v_{2k}}\right)\right]$$

$$= \sigma^{d-1} 2\frac{\phi v_T}{d+2}\left[\int_{\mathbb{R}^d} dv_1 f^{(0)}(v_1)\left(v_{1i}v_{1j} - \frac{\delta_{ij}}{d}v_1^2\right)\underbrace{\int_{\mathbb{R}^d} dv_2\, f^{(1)}(v_2)}_{=0} + \underbrace{\int_{\mathbb{R}^d} dv_1\, f^{(0)}(v_1)}_{=n}\int_{\mathbb{R}^d} dv_2 f^{(1)}(v_2)\left(v_{2i}v_{2j} - \frac{1}{d}v_2^2\delta_{ij}\right)\right]$$

$$= \sigma^{d-1} 2\frac{\phi v_T}{d+2}\frac{n}{m}\int_{\mathbb{R}^d} dv_2\, f^{(1)}(v_2)\,\underbrace{m\left(v_{2i}v_{2j} - \frac{1}{d}v_2^2\delta_{ij}\right)}_{= D_{ij}(v_2)}$$

$$= \sigma^{d-1} 2\frac{\phi v_T}{d+2}\frac{n}{m}\left[P_{ij}^{(1)} - \frac{n}{\beta}\delta_{ij}\right]$$

; $\frac{n}{\beta} = nk_BT = p^{(1)} = 0$: c'est la pression à l'ordre 1 qui est nulle : différent de la pression hydrostatique ! Car: $\int dv\, f^{(1)}(v)\, v^2 = 0$

$$= \sigma^{d-1} 2\frac{\phi v_T}{d+2}\frac{n}{m} P_{ij}^{(1)}$$

Ainsi:

$$\boxed{\int_{\mathbb{R}^d} dv\, m\, v_i v_j L_c[f^{(0)}, f^{(1)}] = \sigma^{d-1} 2\frac{\phi n v_T}{d+2} P_{ij}^{(1)}} \qquad (3)$$

Calcul de l'intégrale d'annihilation:

$$\int_{\mathbb{R}^d} dv\, v_i v_j L_a[f^{(0)}, f^{(1)}] = -\int_{\mathbb{R}^d} dv_1\, v_{1i}v_{1j}\left[-\sigma^{d-1}\phi v_T f^{(1)}(\underline{r}, v_1; t)\int d\underline{v}_2 f^{(0)}(\underline{r}, \underline{v}_2; t) - \sigma^{d-1}\phi v_T f^{(0)}(\underline{r}, \underline{v}_1; t)\int dv_2 f^{(1)}(\underline{r}, v_2; t)\right]$$

$$= \sigma^{d-1}\phi v_T \underbrace{\int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\, f^{(1)}(\underline{r}, \underline{v}_1; t) f^{(0)}(\underline{r}; v_2; t)\, v_{1i}v_{1j}}_{v_1 \leftrightarrow v_2} + \sigma^{d-1}\phi v_T\int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\, f^{(0)}(\underline{r}, \underline{v}_1; t) f^{(1)}(\underline{r}, v_2; t)\, v_{1i}v_{1j}$$

$$= \sigma^{d-1}\phi v_T\int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\, f^{(0)}(\underline{r}, \underline{v}_1; t) f^{(1)}(\underline{r}, v_2; t)\left[v_{1i}v_{1j} + v_{2i}v_{2j}\right]$$

$$= \sigma^{d-1}\phi v_T\int_{\mathbb{R}^d} d\underline{v}_1\, f^{(0)}(\underline{r}, \underline{v}_1; t)\, v_{1i}v_{1j}\underbrace{\int_{\mathbb{R}^d} d\underline{v}_2\, f^{(1)}(\underline{r}, \underline{v}_2; t)}_{=0} + \sigma^{d-1}\phi v_T\underbrace{\int_{\mathbb{R}^d} d\underline{v}_1 f^{(0)}(\underline{r}, \underline{v}_1; t)}_{=n}\int_{\mathbb{R}^d} dv_2\, f^{(1)}(\underline{r}, \underline{v}_2; t)\, v_{2i}v_{2j}$$

$$= \sigma^{d-1}\phi v_T n\int_{\mathbb{R}^d} dv_2\, f^{(1)}(\underline{r}, v_2; t)\, v_{2i}v_{2j}$$

$$= \sigma^{d-1}\phi v_T n \frac{1}{m} P_{ij}^{(1)}$$

Ainsi:

$$\boxed{\int_{\mathbb{R}^d} dv\, m\, v_i v_j L_a[f^{(0)}, f^{(1)}] = \sigma^{d-1}\phi v_T n P_{ij}^{(1)}} \qquad (4)$$

En conclusion, l'Eq. (1) devient:

$$\boxed{\begin{aligned} &\left[\partial_t^{(0)} + p\nu_a + (1-p)\nu_c\right] P_{ij}^{(1)} = -p^{(0)}\Delta_{ijkl}\nabla_k u_l \\ &\nu_a = \sigma^{d-1}\phi v_T n = \zeta_n^{(0)} \\ &\nu_c = \sigma^{d-1}\phi v_T n\frac{2}{d+2} = \frac{2}{d+2}\zeta_n^{(0)} \end{aligned}} \qquad (5)$$

Cohérence avec Santos ?

$$\overset{\text{Santos}}{V_c} = \omega \frac{2}{d+2} \quad ; \; \omega = n\sigma^{d-1} \phi \, v_T \quad \Longrightarrow \quad V_c = n\sigma^{d-1}\phi \, v_T \frac{2}{d+2} \; = \; \frac{2}{d+2} \xi_1^{(0)} \quad : \underline{\text{ok}}.$$

Solution de l'Eq. (5) : elle doit forcément être de la forme

$$\begin{aligned} P_{ij}^{(1)} &= -\eta' \Delta_{ijkl} \nabla_k u_l \\ &= -\eta' \left(\delta_{ik}\delta_{jl} + \delta_{jk}\delta_{il} - \frac{2}{d} \delta_{ij}\delta_{kl} \right) \nabla_k u_l \\ &= -\eta' \left(\nabla_i u_j + \nabla_j u_i - \frac{2}{d} \delta_{ij} \nabla_k u_k \right) \end{aligned}$$

Par identification avec l'expression de la pression

$$P_{ij}(\underline{r},t) = p^{(0)}\delta_{ij} - \eta\left(\nabla_i u_j + \nabla_j u_i - \frac{2}{d}\delta_{ij}\nabla_k u_k\right) - \xi \, \delta_{ij} \nabla_k u_k$$

on en déduit que $\eta' = \eta$ est la viscosité (de cisaillement). Par analyse de dépendance fonctionnelle, il faut que $\eta \propto T^{1/2}$, donc :

$$\begin{aligned} \partial_t P_{ij}^{(1)}(\underline{r},t) &= -\Delta_{ijkl} \nabla_k u_l \;\, \text{cte} \; \frac{1}{2} \left(\partial_t^{(0)} T\right) T^{-1/2} \qquad , \; \eta = \text{cte} \, T^{1/2} \\ &= \underbrace{-\Delta_{ijkl}\nabla_k u_l \left(\text{cte}\, T^{1/2}\right)}_{= P_{ij}^{(1)}(\underline{r},t)} \frac{1}{T} \underbrace{\partial_t^{(0)} T}_{=0 \text{ par l'Eq. de bilan à l'ordre zéro}} \\ &= 0 \end{aligned}$$

Ainsi l'Eq. (5) donne :

$$\left[p V_a + (1-p) V_c \right] \left(-\eta \Delta_{ijkl} \nabla_k u_l \right) = -p^{(0)} \Delta_{ijkl} \nabla_k u_l$$

$$\Longrightarrow \quad \eta = p^{(0)} \frac{1}{pV_a + (1-p)V_c}$$

et :

$$\begin{aligned} \eta^* = \frac{\eta}{\eta_0} &= n k_B T \frac{1}{\xi_1^{(0)}\left[p + \frac{2}{d+2}(1-p)\right]} \frac{1}{\eta_0} \\ &= \frac{n k_B T}{\xi_1^{(0)}} \frac{1}{\eta_0} \frac{d+2}{2} \frac{1}{p\frac{d+2}{2} + (1-p)} \end{aligned}$$

$$; \; \eta_0 = \frac{d+2}{8} \frac{\Gamma(d/2)}{\pi^{(d-1)/2}} \frac{\sqrt{m k_B T}}{\sigma^{d-1}}$$

$$\xi_1^{(0)} = \sigma^{d-1} n \phi \, v_T$$

$$\phi = \frac{4}{\sqrt{2}} \frac{\pi^{(d-1)/2}}{\Gamma(d/2)}$$

$$\begin{aligned} &= \frac{n k_B T}{\cancel{\sigma^{d-1}} n \phi v_T} \frac{\cancel{d+2}}{2} \frac{8}{\cancel{d+2}} \frac{\pi^{(d-1)/2}}{\Gamma(d/2)} \frac{\cancel{\sigma^{d-1}}}{\sqrt{m k_B T}} \frac{1}{p\frac{d+2}{2} + (1-p)} \\ &= \frac{\cancel{n} k_B T}{\cancel{n} v_T} \frac{\sqrt{2}}{\cancel{4}} \frac{\cancel{\Gamma(d/2)}}{\cancel{\pi^{(d-1)/2}}} \cancel{4} \frac{\cancel{\pi^{(d-1)/2}}}{\cancel{\Gamma(d/2)}} \frac{1}{\sqrt{k_B T}} \frac{1}{\sqrt{m}} \frac{1}{p\frac{d+2}{2} + (1-p)} \\ &= \cancel{\sqrt{k_B T}} \sqrt{\frac{\cancel{\beta m}}{\cancel{2}}} \cancel{\sqrt{2}} \frac{1}{\cancel{\sqrt{m}}} \frac{1}{p\frac{d+2}{2} + (1-p)} \end{aligned}$$

$$\boxed{\eta^* = \frac{1}{p\,\frac{d+2}{2} + (1-p)}}$$

Ok : le même résultat que par Chapman-Enskog (à part le facteur 1/2).

- Intégration de ⊛ sur $\underline{V}$ avec poids $\frac{1}{2} m V^2 V_i$:

$$\underbrace{\partial_t^{(0)} \int_{\mathbb{R}^d} d\underline{v} \, \frac{1}{2} m V^2 V_i f^{(1)}}_{= q_i^{(1)}(\underline{r},t)} + \int_{\mathbb{R}^d} d\underline{v} \, \frac{1}{2} m V^2 V_i \, J f^{(1)} = \int_{\mathbb{R}^d} d\underline{v} \, \frac{1}{2} m V^2 V_i A_k(\underline{V}) \nabla_k \ln T + \int_{\mathbb{R}^d} d\underline{v} \, \frac{1}{2} m V^2 V_i B_k(\underline{V}) \nabla_k \ln n + \underbrace{\int_{\mathbb{R}^d} d\underline{v} \, \frac{1}{2} m V^2 V_i \underbrace{C_{kl}(\underline{V})}_{= C_{kl}(-\underline{V})} \nabla_k u_l}_{=0}$$

$$\Longrightarrow \boxed{\partial_t^{(0)} q_i^{(1)}(\underline{r},t) + p \int_{\mathbb{R}^d} dv \, \frac{1}{2} m V^2 V_i L_a[f^{(0)}, f^{(1)}] + (1-p) \int_{\mathbb{R}^d} dv \, \frac{1}{2} m V^2 V_i L_c[f^{(0)}, f^{(1)}] = \int_{\mathbb{R}^d} dv \, \frac{1}{2} m V^2 V_i A_k(\underline{V}) \nabla_k \ln T + \int_{\mathbb{R}^d} dv \, \frac{1}{2} m V^2 V_i B_k(\underline{V}) \nabla_k \ln n} \quad (6)$$

Calcul du membre de droite avec $A_k(\underline{v}) = -v_k T \frac{\partial f^{(0)}}{\partial T} - \frac{k_B T}{m}\frac{\partial f^{(0)}}{\partial v_k}$; $B_k(\underline{v}) = -v_k f^{(0)} - \frac{k_B T}{m}\frac{\partial f^{(0)}}{\partial v_k}$. Ainsi:

$$\int_{\mathbb{R}^d} dv\, v^2 v_i A_k(\underline{v}) = -\int_{\mathbb{R}^d} d\underline{v}\, v^2 v_i v_k T \frac{\partial f^{(0)}}{\partial T} - \frac{k_B T}{m}\int_{\mathbb{R}^d} d\underline{v}\, v^2 v_i \frac{\partial f^{(0)}}{\partial v_k} \quad ; \frac{\partial f^{(0)}}{\partial T} = \frac{\partial f^{(0)}}{\partial v_T}\frac{\partial v_T}{\partial T} \quad ; v_T = \sqrt{\frac{2k_BT}{m}}$$

$$= -\int_{\mathbb{R}^d} d\underline{v}\, v^2 v_i v_k T \frac{\partial v_T}{\partial T}\frac{\partial}{\partial v_T}\left(\frac{1}{\pi^{d/2}}\frac{n}{v_T^d} e^{-v^2/v_T^2}\right) + \frac{k_BT}{m}\int_{\mathbb{R}^d} d\underline{v}\, \frac{\partial}{\partial v_k}(v^2 v_i) f^{(0)}(v) \underbrace{- \frac{k_BT}{m} v^2 v_i \frac{\partial f^{(0)}}{\partial v_k}\Big|_{\partial\Omega}}_{=0}$$

$$= -\int_{\mathbb{R}^d} d\underline{v}\, v^2 v_i v_k T\, \frac{1}{2}\frac{1}{T} v_T \left[f^{(0)}(v)\left(\frac{-d}{v_T}\right) - \left(\frac{\partial}{\partial v_T}\frac{v^2}{v_T^2}\right) f^{(0)}(v)\right] + \frac{k_BT}{m}\int_{\mathbb{R}^d} d\underline{v}\, \frac{\partial}{\partial v_k}(v_j v_j v_i) f^{(0)}(v)$$

$$= -\int_{\mathbb{R}^d} d\underline{v}\, v^2 v_i v_k \frac{1}{2} v_T f^{(0)}(v)\left[-\frac{d}{v_T} - v^2(-2)\frac{1}{v_T^3}\right] + \frac{k_BT}{m}\int_{\mathbb{R}^d} d\underline{v}\, f^{(0)}(\underline{v})\left[\delta_{jk} v_j v_i + v_j \delta_{jk} v_i + v^2 \delta_{ik}\right]$$

$$= -\int_{\mathbb{R}^d} d\underline{v}\, v^2 v_i v_k \frac{1}{2}\cancel{v_T} f^{(0)}(v) \frac{1}{\cancel{v_T}}\left[-d + 2\left(\frac{v}{v_T}\right)^2\right] + \frac{k_BT}{m}\int_{\mathbb{R}^d} d\underline{v}\, f^{(0)}(v)\left[2 v_k v_i + v^2 \delta_{ik}\right]$$

$$= -\frac{1}{2}\int_{\mathbb{R}^d} d\underline{v}\, v^2 v_i v_k \frac{1}{\pi^{d/2}}\frac{n}{v_T^d} e^{-v^2/v_T^2}\left(-d + 2 v^2/v_T^2\right) + \frac{k_BT}{m}\left[2\int_{\mathbb{R}^d} dv\, f^{(0)}(v) v_k v_i + \delta_{ik}\int_{\mathbb{R}^d} dv\, f^{(0)}(v) v^2\right]$$

$$= -\frac{1}{2}\cancel{v_T^d}\int_{\mathbb{R}^d} dc\, v_T^4 c^2 c_i c_k \frac{1}{\pi^{d/2}}\frac{n}{\cancel{v_T^d}} e^{-c^2}\left(-d + 2c^2\right) + \frac{k_BT}{m}\left[2\frac{1}{\pi^{d/2}}\frac{n}{\cancel{v_T^d}}\cancel{v_T^d}\int_{\mathbb{R}^d} dc\, e^{-c^2} c_k c_i v_T^2\right.$$

$$\left. + \delta_{ik}\frac{1}{\pi^{d/2}}\frac{n}{\cancel{v_T^d}}\cancel{v_T^d}\int_{\mathbb{R}^d} dc\, e^{-c^2} v_T^2 c^2\right]$$

$$= -\frac{v_T^4 n}{2\pi^{d/2}}\int_{\mathbb{R}^d} dc\, e^{-c^2} c^2 c_i c_k\left(-d + 2c^2\right) + \underbrace{\frac{k_BT}{m}}_{= (2/\beta m)\frac{1}{2} = v_T^2/2}\left[\frac{2n v_T^2}{\pi^{d/2}}\int_{\mathbb{R}^d} dc\, e^{-c^2} c_i c_k + \delta_{ik}\frac{n v_T^2}{\pi^{d/2}}\int_{\mathbb{R}^d} dc\, e^{-c^2} c^2\right]$$

$$= -\frac{n v_T^4}{2\pi^{d/2}}\left[-d\int_{\mathbb{R}^d} dc\, e^{-c^2} c^2 c_i c_k + 2\int_{\mathbb{R}^d} dc\, e^{-c^2} c^4 c_i c_k\right] + \frac{v_T^2}{2}\frac{n v_T^2}{\pi^{d/2}}\left[2\int_{\mathbb{R}^d} dc\, e^{-c^2} c_i c_k + \delta_{ik}\int_{\mathbb{R}^d} dc\, e^{-c^2} c^2\right]$$

$$= -\frac{n v_T^4}{2\cancel{\pi^{d/2}}}\left[-d\,\cancel{\pi^{d/2}}\frac{d+2}{2d}\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)}\delta_{ik} + 2\cancel{\pi^{d/2}}\frac{d+4}{2d}\frac{\Gamma(\frac{d+4}{2})}{\Gamma(d/2)}\delta_{ik}\right] + \frac{n v_T^4}{2\cancel{\pi^{d/2}}}\left[2\cancel{\pi^{d/2}}\frac{d}{2d}\frac{\Gamma(d/2)}{\Gamma(d/2)}\delta_{ik} + \delta_{ik}\cancel{\pi^{d/2}}\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)}\right]$$

$$= \frac{n v_T^4}{2}\delta_{ik}\left[d\frac{d+2}{2\cancel{d}}\frac{\cancel{d}}{2} - \cancel{2}\frac{d+4}{2\cancel{d}}\frac{d+2}{\cancel{2}}\frac{\cancel{d}}{2} + 2\frac{\cancel{d}}{2\cancel{d}} + \frac{d}{2}\right]$$

$$= \frac{n v_T^4}{2}\delta_{ik}\left[\frac{d(d+2)}{4} - \frac{(d+2)(d+4)}{4} + 1 + \frac{d}{2}\right]$$

$$= \frac{n v_T^4}{8}\delta_{ik}\left[\cancel{d^2} + \cancel{2d} - \cancel{d^2} - 4d - \cancel{2d} - 8 + 4 + 2d\right]$$

$$= \frac{n v_T^4}{8}\delta_{ik}\left[-2d - 4\right]$$

$$= -\frac{n v_T^4}{4}\delta_{ik}(d+2)$$

$\Rightarrow$

$$\int_{\mathbb{R}^d} d\underline{v}\, \frac{1}{2} m v^2 v_i A_k(\underline{v}) \nabla_k \ln T = -\frac{1}{2} m \frac{n}{4}\left(\frac{2}{\beta m}\right)^2 (d+2)\, \delta_{ik} \nabla_k \ln T$$

$$= -\frac{1}{\cancel{2}}\cancel{m}\frac{n}{4}\frac{\cancel{2} k_B T}{\cancel{m}}\Big|^{=p^{(0)}} v_T^2 (d+2)\, \nabla_i \ln T$$

$$= -p^{(0)} v_T^2 \frac{d+2}{4}\nabla_i \ln T$$

$$= -p^{(0)}\frac{2k_B\cancel{T}}{m}\frac{d+2}{4}\frac{1}{\cancel{T}}\nabla_i T$$

$$= \boxed{-\frac{d+2}{2}\frac{p^{(0)} k_B}{m}\nabla_i T}$$

ok.

Calcul du membre de droite :

$$\int_{\mathbb{R}^d} d\underline{v}\, v^2 v_i B_k(\underline{v}) = \int_{\mathbb{R}^d} dv\, v^2 v_i \left[-v_k f^{(0)} - \frac{k_B T}{m} \frac{\partial f^{(0)}}{\partial v_k} \right]$$

$$= -\int_{\mathbb{R}^d} d\underline{v}\, v^2 v_i v_k f^{(0)} - \frac{1}{2}\frac{2}{\beta m}\int_{\mathbb{R}^d} d\underline{v}\, v^2 v_i \frac{\partial f^{(0)}}{\partial v_k}$$

$$= -\frac{1}{\pi^{d/2}}\frac{n}{\cancel{v_T^d}}\cancel{v_T^d}\, v_T^4 \int_{\mathbb{R}^d} dc\, c^2 c_i c_k e^{-c^2} - \frac{1}{2} v_T^2 \left[-\int_{\mathbb{R}^d} d\underline{v} \frac{\partial}{\partial v_k}(v^2 v_i) f^{(0)} + \underbrace{\int_{\partial\Omega} v^2 v_i f^{(0)}}_{=0} \right]$$

$$= -\frac{1}{\cancel{\pi^{d/2}}} n v_T^4 \cancel{\pi^{d/2}} \frac{d+2}{2d} \frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)} \delta_{ik} + \frac{1}{2} v_T^2 \frac{1}{\pi^{d/2}} \frac{n}{\cancel{v_T^d}} v_T^2 \int_{\mathbb{R}^d} dc\, e^{-c^2} \left[2 c_i c_j \delta_{jk} + c^2 \delta_{ik} \right] \cancel{v_T^d}$$

$$= -n v_T^4 \frac{d+2}{2d} \frac{d}{2} \delta_{ik} + n v_T^4 \frac{1}{2\pi^{d/2}} \left[2\int_{\mathbb{R}^d} dc\, e^{-c^2} c_i c_j \delta_{jk} + \int_{\mathbb{R}^d} dc\, e^{-c^2} c^2 \delta_{ik} \right]$$

$$= -n v_T^4 \frac{d+2}{2\cancel{d}} \frac{\cancel{d}}{2} \delta_{ik} + n v_T^4 \frac{1}{2\pi^{d/2}} \left[\cancel{2}\, \pi^{d/2} \frac{\cancel{d}}{\cancel{2d}} \frac{\Gamma(d/2)}{\Gamma(d/2)} \delta_{jk}\delta_{ij} + \pi^{d/2} \frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)} \delta_{ik} \right]$$

$$= -n v_T^4 \frac{d+2}{4} \delta_{ik} + n v_T^4 \frac{1}{2\cancel{\pi^{d/2}}} \cancel{\pi^{d/2}}\, \delta_{ik} \left[1 + \frac{d}{2} \right]$$

$$= \frac{n v_T^4}{2} \delta_{ik} \left[-\frac{d+2}{2} + 1 + \frac{d}{2} \right]$$

$$= \frac{n v_T^4}{2} \delta_{ik} \left[-\frac{d+2}{2} + \frac{d+2}{2} \right]$$

$$\boxed{= 0}$$

Ceci est en accord avec le résultat de Santos, où on a nullité dans le cas $a_2 = 0$ (notre cas).

Calcul de l'intégrale d'annihilation :

$$\int_{\mathbb{R}^d} dv\, v^2 v_i L_a[f^{(0)}, f^{(1)}] = \sigma^{d-1} \varnothing v_T \int_{\mathbb{R}^d} dv_1 f^{(0)}(\underline{r}, \underline{v}_1; t) v_1^2 v_{1i} \underbrace{\int_{\mathbb{R}^d} dv_2 f^{(1)}(\underline{r}, \underline{v}_2; t)}_{=0} + \sigma^{d-1} \varnothing v_T \underbrace{\int_{\mathbb{R}^d} dv_1 f^{(0)}(\underline{r}, \underline{v}_1; t)}_{=n} \int_{\mathbb{R}^d} dv_2 f^{(1)}(\underline{r}, \underline{v}_2; t) v_2^2 v_{2i}$$

$$= \sigma^{d-1} \varnothing v_T\, n \frac{2}{m} \underbrace{\int_{\mathbb{R}^d} d\underline{v}\, f^{(1)}(\underline{r}, \underline{v}; t) \frac{1}{2} m v^2 v_i}_{= q_i^{(1)}(\underline{r}, t)}$$

$$\Longrightarrow \boxed{\int_{\mathbb{R}^d} dv\, \frac{1}{2} m v^2 v_i L_a[f^{(0)}, f^{(1)}] = \overbrace{\sigma^{d-1} \varnothing v_T n}^{= \xi_n^{(0)}}\, q_i^{(1)}(\underline{r}, t)}$$

Calcul de l'intégrale de collision :

$$\int_{\mathbb{R}^d} dv\, v^2 v_i L_c[f^{(0)}, f^{(1)}] \overset{\text{lemme 3.4}}{=} -\sigma^{d-1} \frac{\varnothing v_T}{\sqrt{d}} \int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\, f^{(0)}(\underline{v}_1) f^{(1)}(\underline{r}, \underline{v}_2; t) \int d\hat{\sigma}\, (b-1) \left[v_1^2 v_{1i} + v_2^2 v_{2i} \right]$$

Avec :

$$(b-1)\left[v_1^2 v_{1i} + v_2^2 v_{2i} \right] = (\underline{g}\cdot\hat{\sigma})^2 (v_{1j} + v_{2j})(\delta_{ij} + 2\sigma_i \sigma_j) - (\underline{g}\cdot\hat{\sigma})(v_1^2 \delta_{ij} + 2 v_{1i} v_{1j} - v_2^2 \delta_{ij} - 2 v_{2i} v_{2j}) \sigma_j$$

$$\Rightarrow \int d\hat{\sigma} (b-1)\left[v_1^2 v_{1i} + v_2^2 v_{2i} \right] = (v_{1j} + v_{2j}) \left[\delta_{ij}\, 2\beta_2\, g^2 + 2 \frac{2\beta_2}{d+2} (2 g_i g_j + g^2 \delta_{ij}) \right] - (v_1^2 \delta_{ij} + 2 v_{1i} v_{1j} - v_2^2 \delta_{ij} - 2 v_{2i} v_{2j})\, 2\beta_2\, g_j$$

$$= (v_{1j} + v_{2j}) \frac{4\beta_2}{d+2}\, 2\,(v_{1i} v_{1j} - v_{1i} v_{2j} - v_{2i} v_{1j} + v_{2i} v_{2j}) + (v_{1j} + v_{2j}) \frac{4\beta_2}{d+2} \delta_{ij} (v_1^2 + v_2^2 - 2 v_{1k} v_{2k})$$

0 : parité intégr. sur $\underline{v}_1$

$$- 2\beta_2 \left(v_1^2 \cancelto{0}{v_{1j}} \delta_{ij} + 2 v_{1i} v_{1j} \cancelto{0}{v_{1j}} - v_2^2 \cancelto{0}{v_{1j}} \delta_{ij} - 2 v_{2i} v_{2j} \cancelto{0}{v_{1j}} \right) + (v_{1j} + v_{2j})\, 2\beta_2\, \delta_{ij} (v_1^2 + v_2^2 - 2 v_{1k} v_{2k})$$

$$+ 2\beta_2 \left(v_1^2 v_{2j} \delta_{ij} + 2 v_{1i} v_{1j} v_{2j} - v_2^2 v_{2j} \delta_{ij} - 2 v_{2i} v_{2j} v_{2j} \right)$$

$$= \frac{8\beta_2}{d+2} \left(v_{1i} \cancelto{0}{v_{1j}} v_{1j} - v_{1i} v_{2j} v_{1j} - v_{2i} v_{1j} v_{1j} + v_{2i} \cancelto{0}{v_{2j}} v_{1j} \right)$$

$$+ \frac{8\beta_2}{d+2} \left(v_{1i} v_{1j} v_{2j} - v_{1i} \cancelto{0}{v_{2j}} v_{2j} - v_{2i} \cancelto{0}{v_{1j}} v_{2j} + v_{2i} v_{2j} v_{2j} \right)$$

$$+ \frac{4\beta_2}{d+2} \delta_{ij} \left(\cancelto{0}{v_1^2 v_{1j}} + v_2^2 v_{2j} - 2 v_{1k} v_{2k} v_{1j} + v_1^2 v_{2j} + v_2^2 v_{2j} - 2 v_{1k} \cancelto{0}{v_{2k}} v_{2j} \right)$$

$$+ 2\beta_2 \delta_{ij} \left(\cancelto{0}{v_1^2 v_{1j}} + v_2^2 v_{1j} - 2 v_{1k} v_{2k} v_{1j} + v_1^2 v_{2j} + v_2^2 v_{2j} - 2 v_{1k} \cancelto{0}{v_{2k}} v_{2j} \right)$$

$$+ 2\beta_2 \left(v_1^2 v_{2i} + 2 v_{1i} v_{1j} v_{2j} - v_2^2 v_{2i} - 2 v_{2i} v_{2j} v_{2j} \right)$$

$$= \frac{8\beta_2}{d+2}\Big[-V_{1i}\cancel{V_{1j}}V_{2j} - V_1^2 V_{2i} + V_{1i}\cancel{V_{1j}}V_{2j} + V_2^2 V_{2i} - V_{1k}V_{2k}V_{1j}\delta_{ij} + \tfrac{1}{2}V_1^2 V_{2j}\delta_{ij} + \tfrac{1}{2}V_2^2 V_{2j}\delta_{ij}\Big]$$

$$+ 2\beta_2\Big[V_2^2 V_{1j}\delta_{ij} - 2V_{1k}V_{2k}V_{1j}\delta_{ij} + V_1^2 V_{2j}\delta_{ij} + V_2^2 V_{2j}\delta_{ij} + V_1^2 V_{2i} + 2V_{1i}V_{1j}V_{2j} - V_2^2 V_{2i} - 2V_{2i}V_{2j}V_{2j}\Big]$$

$$= \frac{8\beta_2}{d+2}\Big[-V_1^2 V_{2i} + V_2^2 V_{2i} - V_{1k}V_{2k}V_{1i} + \tfrac{1}{2}V_1^2 V_{2i} + \tfrac{1}{2}V_2^2 V_{2i}\Big]$$

$$+ 2\beta_2\Big[\cancel{V_2^2 V_{1i}}^{\,0} - 2V_{1k}\cancel{V_{2k}}V_{1i} + V_1^2 V_{2i} + \cancel{V_2^2 V_{2i}} + V_1^2 V_{2i} + 2V_{1k}\cancel{V_{2k}}V_{1i} - \cancel{V_2^2 V_{2i}} - 2V_2^2 V_{2i}\Big]$$

$$= \frac{8\beta_2}{d+2}\Big[-\tfrac{1}{2}V_1^2 V_{2i} + \tfrac{3}{2}V_2^2 V_{2i} - V_{1k}V_{2k}V_{1i}\Big] + 2\beta_2\Big[\cancel{V_2^2 V_{1i}}^{\,0} + V_1^2 V_{2i}\,2 - 2V_2^2 V_{2i}\Big]$$

$$= V_1^2 V_{2i}\, 4\beta_2\Big[1 + \frac{2}{d+2}\Big] + V_2^2 V_{2i}\, 4\beta_2\Big[\underbrace{-1 + \frac{3}{d+2}}_{\frac{3-d-2}{d+2} = \frac{1-d}{d+2}}\Big] - V_{1k}V_{2k}V_{1i}\frac{8\beta_2}{d+2}$$

$$= 4\beta_2 \frac{d+4}{d+2} V_1^2 V_{2i} - 4\beta_2 \frac{d-1}{d+2} V_2^2 V_{2i} - V_{1k}V_{2k}V_{1i}\frac{8\beta_2}{d+2}$$

⋮

etc.

à revérifier.

En utilisant le résultat de Santos on a:

$$\boxed{\int_{\mathbb{R}^d} dv\, \tfrac{1}{2} m V^2 V_i\, L_c[f^{(0)}, f^{(1)}] = \frac{d-1}{d}\,\frac{2}{d+2}\, n\sigma^{d-1}\varnothing V_T\, q_i^{(1)}(\underline{r},t)}$$

On met tout ensemble dans (6) pour obtenir:

$$\Big[\partial_t^{(0)} + p\xi_n^{(0)} + (1-p)\frac{2(d-1)}{d(d+2)}\xi_n^{(0)}\Big] q_i^{(1)}(\underline{r},t) = -\frac{d+2}{2}\frac{p^{(0)}k_B}{m}\nabla_i T$$

Le flux de chaleur est de la forme

$$q_i^{(1)}(\underline{r},t) = -\lambda\nabla_i T - \mu\nabla_i n$$

car $\mu \sim \mu_0 = \frac{T\lambda_0}{n}$

Par analyse de dépendence fonctionnelle, il faut que $\lambda \propto T^{1/2}$; $\mu \propto T^{3/2} n^{-1}$, et donc:

$$\partial_t^{(0)} q_i^{(1)}(\underline{r},t) = -\nabla_i T\,\partial_t^{(0)}\big(\text{cte}\,T^{1/2}\big) - \nabla_i n\,\partial_t^{(0)}\big(\text{cte}'\,n^{-1}T^{3/2}\big)$$

$$= -\nabla_i T\underbrace{\big(\text{cte}\,T^{1/2}\big)}_{=\lambda}\frac{1}{2}\frac{1}{T}\underbrace{\partial_t^{(0)}T}_{=0} - \nabla_i n\underbrace{\big(\text{cte}'\,T^{3/2}\big)}_{=\mu}\frac{3}{2}\frac{1}{T}\underbrace{\partial_t^{(0)}T}_{=0} - \nabla_i n\big(\text{cte}'\,T^{3/2}\big)\underbrace{\big(\partial_t^{(0)}n^{-1}\big)}_{\substack{=-\frac{1}{n}\frac{1}{n}\partial_t^{(0)}n \\ =+\frac{1}{n}\xi_n^{(0)}p}}$$

$$= 0 + \mu\nabla_i n\,(p\,\xi_n^{(0)})$$

Ainsi:

$$\Big[p\,\xi_n^{(0)} + (1-p)\frac{2(d-1)}{d(d+2)}\xi_n^{(0)}\Big]\big(-\lambda\nabla_i T - \mu\nabla_i n\big) + \mu\nabla_i n\, p\,\xi_n^{(0)} = -\frac{d+2}{2}\frac{p^{(0)}k_B}{m}\nabla_i T$$

$\Longrightarrow \mu = 0$; car: $\mu\nabla_i n\,\xi_n^{(0)} = \mu\nabla_i n\,(1-p)\frac{2(d-1)}{d(d+2)}\xi_n^{(0)} \Rightarrow (1-p)\frac{2(d-1)}{d(d+2)} = 1\ \forall p, d \Longrightarrow \mu = 0$

$$\lambda = \frac{d+2}{2}\frac{p^{(0)}k_B}{m}\frac{1}{p\,\xi_n^{(0)} + (1-p)\frac{2(d-1)}{d(d+2)}\xi_n^{(0)}}$$

$$= \frac{d+2}{2}\frac{p^{(0)}k_B}{m}\frac{1}{\xi_n^{(0)}}\frac{1}{p + (1-p)\frac{2(d-1)}{d(d+2)}}$$

$$= \frac{d+2}{2}\frac{p^{(0)}k_B}{m}\frac{1}{\xi_n^{(0)}}\frac{d(d+2)}{2(d-1)}\frac{1}{p\frac{d(d+2)}{2(d-1)} + (1-p)}$$

et:

$$\lambda^* = \frac{\lambda}{\kappa_0} = \lambda\,\frac{2(d-1)}{d(d+2)}\frac{m}{k_B}\frac{1}{\zeta_0}$$

$$= \frac{d+2}{2}\frac{p^{(0)}\cancel{k_B}}{\cancel{m}}\frac{1}{\xi_n^{(0)}}\frac{\cancel{d(d+2)}}{\cancel{2(d-1)}}\frac{\cancel{2(d-1)}}{\cancel{d(d+2)}}\frac{\cancel{m}}{\cancel{k_B}}\frac{1}{\zeta_0}\frac{1}{p\frac{d(d+2)}{2(d-1)} + (1-p)}$$

$$= \frac{d+2}{2}\,\rho^{(c)}\,\frac{1}{f_\lambda^{(c)}}\,\frac{8}{d+2}\,\frac{\pi^{(d-1)/2}}{\Gamma(d/2)}\,\frac{\sigma^{d-1}}{\sqrt{m k_B T}}\,\frac{1}{\rho\frac{d(d+2)}{2(d-1)}+(1-\rho)}$$

$$= \frac{\cancel{d+2}}{2}\,\cancel{\rho}\, k_B T\,\frac{1}{\cancel{\sigma^{d-1}}\,\phi\sqrt{1}\,\cancel{\rho}}\,\frac{8}{\cancel{d+2}}\,\frac{\pi^{(d-1)/2}}{\Gamma(d/2)}\,\frac{\cancel{\sigma^{d-1}}}{\sqrt{m k_B T}}\,\frac{1}{\rho\frac{d(d+2)}{2(d-1)}+(1-\rho)} \qquad ;\ \phi = \frac{4}{\sqrt{2}}\,\frac{\pi^{(d-1)/2}}{\Gamma(d/2)}$$

$$= \frac{\cancel{k_B T}}{\cancel{2}}\,\cancel{8}\,\frac{1}{\sqrt{\frac{\cancel{2 k_B T}}{\cancel{m}}}}\,\frac{\cancel{\sqrt{2}}}{\cancel{4}}\,\frac{\cancel{\Gamma(d/2)}}{\cancel{\pi^{(d-1)/2}}}\,\frac{\cancel{\pi^{(d-1)/2}}}{\cancel{\Gamma(d/2)}}\,\frac{1}{\sqrt{\cancel{m k_B T}}}\,\frac{1}{\rho\frac{d(d+2)}{2(d-1)}+(1-\rho)}$$

$$\boxed{\lambda^* = \frac{1}{\rho\frac{d(d+2)}{2(d-1)}+(1-\rho)}}$$

Conclusion:

$$\boxed{\begin{aligned} \eta^* &= \frac{1}{\rho\frac{d+2}{2}+(1-\rho)} \\ \kappa^* &= \frac{1}{\rho\frac{d(d+2)}{2(d-1)}+(1-\rho)} \\ \mu^* &= 0 \end{aligned}}$$